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| CSE 440: Introduction to Artificial Intelligence | Practice Exam - Midterm |
| Instructor: Vishnu Boddeti | Spring 2023 |

Full Name: ______________________ MSU ID: ____________________

**Instructions:** Please read the instructions carefully. Failure to follow any of these instructions will result in a 10 point penalty. There will not be any exceptions.

(1) Write your name on the question paper.
(2) **Attach your cheat sheet to your answers at the bottom. Else you will receive zero points for the exam.**
(3) **Please write legibly to receive full credit.**
(4) **Please show your work and justify your answers to receive partial credit.**

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|---|---|---|
| Q 01: | ____________ | / 25.0 |
| Q 02: | ____________ | / 20.0 |
| Q 03: | ____________ | / 15.0 |
| Q 04: | ____________ | / 15.0 |
| Q 05: | ____________ | / 10.0 |
| Q 06: | ____________ | / 15.0 |
| Total: | ____________ | / 100.0 |

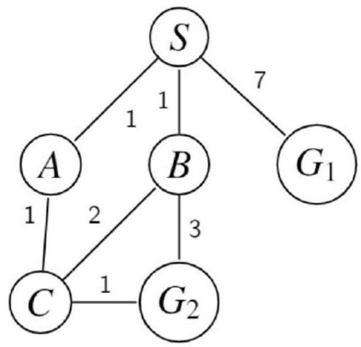



FIGURE 1. State-space graph

(1) (**25 points**) **Multiple Choice Questions:** Select all the answers that you think are correct. These questions may have one or more correct answers. You should circle **all that apply**.

(a) Which of the following statements is/are true given for a heuristic function $h$?

(1) If $h(n) = h^*(n)$ for all $n$, then the algorithm $A^*$ will only expand nodes on the optimal path (ignoring ties).

(2) If $h$ is admissible, the smaller $h(n)$ is, the fewer nodes that $A^*$ will expand.

(3) If $h(n)$ is always less than or equal to the cost of the cheapest path from $n$ to the goal, then $A^*$ is guaranteed to find an optimal solution.

(i) Only (1) is true

(ii) Only (2) is true

(iii) Only (3) is true

(iv) Both (1) and (3) are true

(v) All (1), (2) and (3) are true

(b) Which goal is reached and what is the total cost of the solution found for the state-space graph in figure 1 when using Breadth-First Search and Uniform-Cost Search ( $S$ is the start state, $G_1$ and $G_2$ are the goal states, arc are bidirectional, no repeated state checking, break any ties alphabetically)

(i) BFS: $G_1$ ( Cost:7), UCS: $G_2$ (Cost:4)

(ii) BFS: $G_2$ (Cost:4), UCS: $G_1$( Cost:7).

(iii) BFS: $G_2$ ( Cost:4), UCS: $G_2$( Cost:4).

(iv) BFS: $G_1$ (Cost:7), UCS: $G_2$ (Cost:3).

(v) BFS: $G_1$ (Cost:7), UCS: $G_1$ (Cost:7).

(c) In constraint satisfaction problems, some heuristics can be used to improve backtracking search. These heuristics strategies include:

(i) the "minimum remaining value" strategy to pick which variable to explore

(ii) the "most constraining variable" strategy to pick which variable to explore

(iii) the "least constraining value" strategy to pick which value to assign first to a given variable

(iv) the "least constraining value" strategy to pick which variable to explore

(v) constraint propagation for early detection of conflicts

(d) If $f(s), g(s)$ and $h(s)$ are all admissible heuristics, then which of the following are also guaranteed to be admissible heuristics?

(i) $f(s) + g(s) + h(s)$

(ii) $f(s)/3 + g(s)/3 + h(s)/3$

(iii) $f(s)/6 + g(s)/3 + h(s)/2$

(iv) $f(s) * g(s) * h(s)$

(v) $\min(f(s), g(s), h(s))$

(vi) $\min(f(s), g(s) + h(s))$

(vii) $\max(f(s), g(s), h(s))$

(viii) $\max(f(s), g(s) + h(s))$

(e) Consider a CSP with variable s $X, Y$ with domains $\{1, 2, 3, 4, 5, 6\}$ for $X$ and $\{2, 4, 6\}$ for $Y$, and constraints $X < Y$ and $X + Y > 8$. What are the values that will remain in the domain of $X$ after enforcing arc consistency for the arc $X \rightarrow Y$ (recall arc consistency for a specific arc only prunes the domain of the tail variable, in this case $X$) ?

(i) $\{1, 2, 3\}$

(ii) $\{3, 4, 5\}$

(iii) $\{4, 5, 6\}$

(iv) $\{2, 4, 6\}$

(2) (**15 points**) **Optimization:** Problem Formation: Suppose that $f(x) = h\left(g_1(x), g_2(x), \cdots, g_k(x)\right)$, were $h : \mathcal{R}^k \to R$ is convex, and $g_i : \mathcal{R}^n \to \mathcal{R}$. Suppose that for each $i$,

(a) (**5 points**) $h$ is nondecreasing in the $i-$ th argument, and $g_i$ is convex, is $f$ convex? Why?

(b) (**5 points**) $h$ is nonincreasing in the $i-$ th argument, and $g_i$ is concave, is $f$ convex? Why?

(c) (**5 points**) $g_i$ is a linear function, is $f$ convex? Why?

(3) (**15 points**) **Linear Programming:**

(a) (**5 points**) If x and y satisfy the conditions that:

$$\begin{cases} 2x + y - 2 \geq 0 \\ x - 2y + 4 \geq 0 \\ 3x - y - 3 \leq 0 \end{cases}$$

Then what is the largest and smallest value of $z = x^2 + y^2$ ?

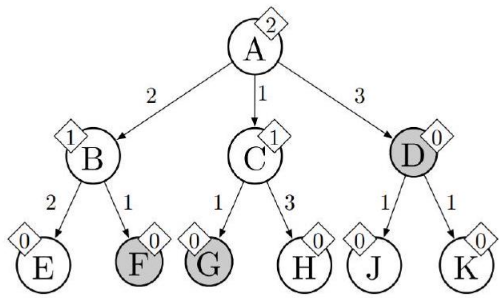
(b) (**10 points**) Let $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$, $C^T = (3, 2, 1, 2, 3)$ and $b = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$.

Consider the corresponding linear programming problem in standard form:

minimize $C^T x$ subject to $Ax = b, x \geq 0$

(i) Determine an optimal solution and the optimal cost.
(ii) Is the optimal solution unique?
(iii) How many basic solutions are there?
(iv) How many basic feasible solutions are there?

(4) (**20 points**) **Search algorithms:**
Consider the state search problem shown below. $A$ is the start state and the shaded states are goal states. Arrows are possible state transitions, and numbers by the arrows are costs of each action. Numbers in diamond shape are heuristic values that are used to estimate the optimal cost from that node to the goal.




For each of the following search algorithms, please write down the nodes removed from the fringe and the path it returned. Assumed that the data structure implementations and successor state ordering are all such that ties are broken alphabetically. For example, a partial plane $S \to X \to A$ would be expanded before $S \to X \to B$.

(a) (**4 points**) Depth First Search (ignores costs)
Nodes removed from fringe:
Path returned:

(b) (**4 points**) Breadth First Search (ignores costs)
Nodes removed from fringe:
Path returned:

(c) (**4 points**) Uniform Cost Search
Nodes removed from fringe:
Path returned:

(d) (**4 points**) Greedy Search
Nodes removed from fringe:
Path returned:

(e) (**4 points**) $A^*$ Search
Nodes removed from fringe:
Path returned:

(5) (**15 points**) **CSP Formulation:**

Nodes $S, B, M, D, P, G$ are lining up next to each other with numbered positions 1,2 , $3, 4, 5, 6$, where 1 neighbors 2, 2 neighbors 1 and 3,3 neighbors 2 and 4,4 neighbors 3 and 5, 5 neighbors 4 and 6 , and 6 neighbors 5 . Each one of them takes up exactly one spot. $B$ needs to be next to M one one side and D on the other side. P needs to be next to the G. S needs to be at 1 or 2 . Formulate this problem as a CSP: list the variables, their domains, and the constraints. Encode unary constraints as a constraint rather than pruning the domain. (You do not need to solve the problem, just provide variables, domains and implicit constraints).

(a) (**5 points**) Variables:

(b) (**5 points**) Domains:

(c) (**5 points**) Constraints:

(6) (**10 points**) **Zero-sum game:**
Consider a zero-sum game with two players, one's goal is to maximize agent and one's goal is to minimize agent. In this game, the ordering of moves is no longer deterministic.Each turn, a coin is flipped in order to determine which agent gets to make a move during that time step.

Consider the game tree below which playing for two turns. It is currently the move from the maximizer, so the top node is a max node. As we don't know which agent is going to play on the next turn, we've replaced those nodes with boxes. Draw a new game tree that consists of only the traditional min, max, and expecti-nodes that models this situation. Then, fill in the values in each boxes and perform $\alpha$ - $\beta$ pruning.